3-7 Videos Guide

3-7a

- Definition of local maximum and local minimum values
  - If $f(a,b) \geq f(x,y)$ for all (x,y) in an open disk, then $f(a,b)$ is a local maximum
  - If $f(a,b) \leq f(x,y)$ for all (x,y) in an open disk, then $f(a,b)$ is a local minimum
- Definition of a critical point
  - A critical point is a point (a,b) in the domain of f such that $f_x(a,b) = f_y(a,b) = 0$ or one of the first partial derivatives does not exist
- Second Derivatives Test and description of a saddle point
  - Let (a,b) be a critical point of f and let $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$
    - If $D < 0$, then (a,b) is a saddle point
    - If $D > 0$, then
      - if $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum
      - if $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum
    - If $D = 0$, then the Second Derivatives Test is inconclusive

3-7b

Exercise:

- Find the local maximum and minimum values and saddle point(s) of the function. Then graph the surface using a window that shows these characteristics.
  $f(x,y) = xye^{-(x^2+y^2)/2}$

3-7c

- Absolute extrema
  - If $f(a,b) \geq f(x,y)$ for all (x,y) in the domain of f, then $f(a,b)$ is the absolute maximum
  - If $f(a,b) \leq f(x,y)$ for all (x,y) in the domain of f, then $f(a,b)$ is the absolute minimum

Exercise:

- Find the absolute maximum and minimum values of f on the set D.
  $f(x,y) = x + y - xy$; D is the closed triangular region with vertices $(0,0)$, $(0,2)$, and $(4,0)$.